Problem E1. The magnetic permeability of water *(10 points)*

The effect of a magnetic field on most of substances other than ferromagnetics is rather weak. This is because the energy density of the magnetic field in substances of relative magnetic permeability μ is given by the formula $w = \frac{B^2}{2\mu\mu_0}$, and typically μ is very close to 1. Still, with suitable experimental techniques such effects are firmly observable. In this problem we study the effect of a magnetic field, created by a permanent neodymium magnet, on water and use the results to calculate the magnetic permeability of water. **You are not asked to estimate any uncertainties throughout this problem and you do not need to take into account the effects of surface tension.**

The setup comprises of 1 a **stand** (the highlighted numbers correspond to the numbers in the fig.), 3 a **digital caliper**, 4 a **laser pointer**, 5 a **water tray** and 7 **a cylindrical permanent magnet** in the water tray (the magnet is axially magnetised). The water tray is fixed to the base of the stand by the magnet's pull. The laser is fixed to the caliper, the base of which is fastened to the stand; the caliper allows horizontal displacement of the laser. The on-off button of the laser can be kept down with the help of 13 the white conical tube. Do not leave the Laser switched on unnecessarily. The depth of the water above the magnet should be reasonably close to 1 mm (if shallower, the water surface becomes so curved that it will be difficult to take readings from the screen). 15 A cup of water and 16 a syringe can be used for the water level adjustment (to raise the level by 1 mm, add 13 ml of water). 2 A sheet of graph paper (the "screen") is to be fixed to the vertical plate with 14 small magnetic tablets. If the laser spot on the screen becomes smeared, check for a dust on the water surface (and blow away).

The remaining legend for the figure is as follows: 6 the point where the laser beam hits the screen; 11 the LCD screen of the caliper, 10 the button which switches the caliper units between millimeters and inches; 8 on-off switch; 9 button for setting the origin of the caliper reading. Beneath the laser pointer, there is one more button on the caliper, which temporarily re-sets the origin (if you pushed it inadvertently, push it once again to return to the normal measuring mode).

Numerical values for your calculations:

Horizontal distance between the magnet's centre and the screen $L_0 = 490\,\mathrm{mm}$. Check (and adjust, if needed) the alignment of the centre of the magnet in two perpendicular directions. The vertical axis of the magnet must intersect with the laser beam, and it must also intersect with 12 the black line on the support plate.

Magnetic induction (magnetic field strength) on the magnet's axis, at a height of 1 mm from the flat surface, $B_0 = 0.50\,\mathrm{T}$

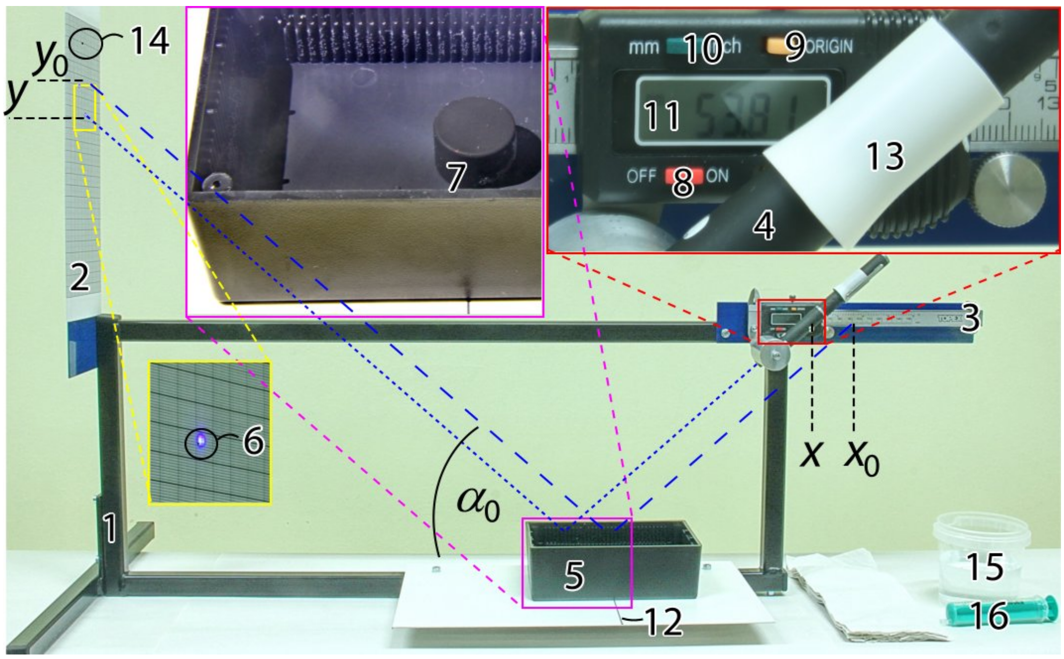
Density of water $\rho_w = 1000\,\mathrm{kg/m^3}$

Acceleration of free-fall $g = 9.8\,\mathrm{m/s^2}$

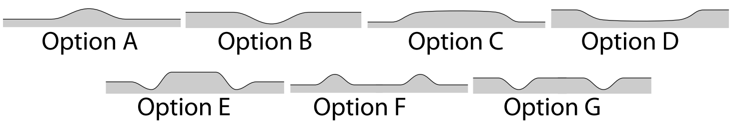
Permeability of a vacuum $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$

WARNINGS:

- **The laser orientation is pre-adjusted, do not move it!**
- **Do not look into the laser beam or its reflections!**
- **Do not try to remove the strong neodymium magnet!**
- **Do not put magnetic materials close to the magnet!**
- **Turn off the laser when not used, batteries drain in 1 h!**



Part A. Qualitative shape of the water surface *(1 points)*
When a cylindrical magnet is placed below water surface, the latter becomes curved. By observation, determine the shape of the water surface above the magnet. Based on this observation, decide if the water is diamagnetic ($\mu < 1$) or paramagnetic ($\mu > 1$).



Write the letter corresponding to the correct option into the Answer Sheet, together with an inequality $\mu > 1$ or $\mu < 1$.

For this part, you do not need to justify your answer.

Part B. Exact shape of the water surface *(7 points)*
Curving of the water surface can be checked with high sensitivity by measuring the reflection of the laser beam from the surface. We use this effect to calculate the dependence of the depth of the water on the horizontal position above the magnet.

i. (1.6 pts) Measure the dependence of the vertical position y of the laser spot on the screen on the caliper reading x (see figure). You should cover the whole usable range of caliper displacements. Write the results into the Table in the Answer Sheet.

ii. (0.7 pts) Draw the graph of the measured dependence.

iii. (0.7 pts) Using the obtained graph, determine the angle α_0 between the beam and the horizontal surface of the water.

iv. (1.4 pts) please note that the slope ($\tan\beta$) of **the water surface** can be expressed as follows:

$$\tan\beta \approx \beta \approx \frac{\cos^2\alpha_0}{2} \cdot \frac{y - y_0 - (x - x_0)\tan\alpha_0}{L_0 + x - x_0},$$

where y_0 is the vertical position of the laser spot on the screen when the beam is reflected from the water surface at the axis of the magnet, and x_0 is the respective position of the caliper. Calculate the values of the slope of the water surface and enter them into the Table on the Answer Sheet. Please note that it may be possible to simplify your calculations if you substitute some combination of terms in the given expression for the slope with a reading from the last graph.

v. (1.6 pts) Calculate the height of the water surface relative to the surface far from the magnet, as a function of x, and write it into the Table on the Answer Sheet.

vi. (1.0 pts) Draw the graph of the latter dependence. Indicate on it the region where the beam hits the water surface directly above the magnet.

Part C. Magnetic permeability *(2 points)*
Using the results of Part B, calculate the value of $\mu - 1$ (the so-called *magnetic susceptibility*), where μ is the relative magnetic permeability of the water. Write your final formula and the numerical result into the Answer Sheet.